

# Nonhomogeneity driven Universe acceleration

Jerzy Stelmach and Izabela Jakacka  
 Institute of Physics, University of Szczecin,  
 Wielkopolska 15, 70-451 Szczecin, Poland

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E-mail: [jerzyst@wmf.univ.szczecin.pl](mailto:jerzyst@wmf.univ.szczecin.pl)  
 and [ijakacka@poczta.onet.pl](mailto:ijakacka@poczta.onet.pl)

## Abstract

Class of spherically symmetric Stephani cosmological models is examined in the context of evolution type. It is assumed that the equation of state at the symmetry center of the models is barotropic ( $p(t) = \alpha\rho(t)$ ) and the function  $k(t)$  playing role of spatial curvature is proportional to Stephani version of the Friedmann-Robertson-Lemaitre-Walker scale factor  $R(t)$  ( $k(t) = \beta R(t)$ ).

Classification of cosmological models is performed depending on different values and signs of parameters  $\alpha$  and  $\beta$ . It is shown that for  $\beta < 0$  (hyperbolic geometry) dust-like ( $\alpha = 0$ ) cosmological model exhibits accelerated expansion at later stages of evolution.

The Hubble and deceleration parameters are defined in the model and it is shown that the deceleration parameter decreases with the distance becoming negative for sufficiently distant galaxies.

Redshift-magnitude relation  $m(z)$  is calculated and discussed in the context of SNIa observational data. It is noticed that the most distant supernovae of type Ia fit quite well to the relation  $m(z)$  calculated in the considered model ( $H_0 = 65$  km/sMpc,  $\Omega_0 \leq 0.3$ ) without introducing the cosmological constant.

It is also shown that the age of the universe in the model is longer than in the Friedmann model corresponding to the same  $H_0$  and  $\Omega_0$  parameters.

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# 1 Introduction

In recent years strong observational evidencies appeared supporting the claim that the universe is accelerating its expansion. The observations are based on supernovae of type Ia (SnIa) used as standard candles in order to calculate the distance to them [1]. Since in standard Friedmann-Robertson-Lemaitre-Walker (FRLW) cosmological models, with nonrelativistic matter and radiation, acceleration is not possible, some people argue that the universe must be dominated by cosmological constant or by some other smooth exotic component influencing negative pressure. There are many different physical scenarios trying to explain existence of such exotic form of matter [2, 3, 4, 5, 6].

In the present paper we propose different approach, in which exotic matter is not necessary to drive accelerated expansion. Instead we relax the assumption of homogeneity of space leaving the isotropy with respect to one point. In other words we replace the usual Cosmological Principle by Copernican Cosmological Principle [7] or some generalized version of the Copernican Principle stating that our Galaxy is at the centre of the Universe. From philosophical point of view this assumption seems to be very artificial but our intention is to show that even such an exotic model does not contradict observations [8].

We shall deal with spherically symmetric Stephani models [9, 10, 11, 12, 13, 14] with the metric given by

$$ds^2 = c^2 \left[ F(\tau) \frac{R(\tau)}{V(r, \tau)} \frac{d}{d\tau} \left( \frac{V(r, \tau)}{R(\tau)} \right) \right]^2 d\tau^2 - \frac{R^2(\tau)}{V^2(r, \tau)} (dr^2 + r^2 d\Omega^2), \quad (1)$$

where the functions  $V(r, \tau)$  and  $F(\tau)$  are defined

$$V(r, \tau) = 1 + \frac{1}{4} k(\tau) r^2, \quad (2)$$

$$F(\tau) = \frac{1}{c} \frac{R(\tau)}{\sqrt{C^2(\tau) R^2(\tau) - k(\tau)}}. \quad (3)$$

The functions  $C(\tau)$ ,  $k(\tau)$  and  $R(\tau)$  are not all independent but are related with each other with the help of expressions

$$\rho(\tau) = \frac{c^4}{8\pi G} 3C^2(\tau), \quad (4)$$

$$p(r, \tau) = \frac{c^4}{8\pi G} \left[ 2C(\tau) \dot{C}(\tau) \frac{V(r, \tau)/R(\tau)}{(V(r, \tau)/R(\tau))} - 3C^2(\tau) \right], \quad (5)$$

following from the Einstein equations.  $(\dot{\phantom{x}})$  denotes a derivative with respect to  $\tau$ . Note that in the considered spherically symmetric Stephani models and in the given coordinate system, the energy density  $\rho(\tau)$  is uniform, while the pressure  $p(r, \tau)$  is not and depends on the distance from the symmetry center placed at  $r = 0$ . This is the reason why in such models barotropic equation of state (i.e. of the form  $p = p(\rho)$ ) does not exist. If we, however, assume some relation between  $\rho(\tau)$  and  $p(r, \tau)$ , this could allow us to eliminate one of the unknown functions, e.g.  $C(\tau)$ . Hence we are left with two unknown functions  $k(\tau)$  and  $R(\tau)$ . The first one  $k(\tau)$  plays a role of spatial curvature index, while the second one  $R(\tau)$  is Stephani version of FRLW scale factor. As far as in FRLW models the curvature index is constant ( $k = 0, \pm 1$ ) and does not change in the process of evolution of the universe, in Stephani models this is not the case. Due to matter transfer, caused by the gradient of pressure, the curvature index may change with time [15]. Similarly as in FRLW cosmology, where the value of  $k$  must be determined from observations, in Stephani models the choice concerns the whole function  $k(\tau)$ . As regards the scale factor  $R(\tau)$ , some arbitrariness is connected with the choice of time parameter  $\tau$ . We remove this arbitrariness by assuming that the time parameter is a proper time of an observer placed at the symmetry center of the model ( $r \approx 0$ ).

In the next section we determine spherically symmetric Stephani cosmological model by assuming barotropic equation of state in the neighbourhood of the symmetry center and by assuming a relation between the scale factor  $R(\tau)$  and arbitrary function  $k(\tau)$ .

In Section 3 we perform classification of possible cosmological models depending on two parameters:  $\alpha$ -parameter appearing in the equation of state, therefore determining kind of matter filling up the universe, and  $\beta$ -parameter responsible for the global geometry of the model.

In Section 4 we focus our attention on one model corresponding to vanishing  $\alpha$ -parameter (dust-like matter) and to negative  $\beta$ -parameter (hyperbolic geometry). The model exhibits accelerated expansion at later stages of evolution.

Section 5 is devoted to definition of Hubble and deceleration parameters in spherically symmetric Stephani models. We show that in the considered model, in spite of the fact that local observations of galaxies give positive deceleration parameter, in the case of more distant ones, negative deceleration parameter can be observed.

In Section 6 we discuss the redshift-magnitude relation in the model and

fit our theoretical curves to the supernovae Ia observational data.

In Section 7 we compare age of the universe in the considered Stephani model to the age in FLRW model determined by the same  $H_0$ ,  $\Omega_0$  and  $\alpha$  parameters.

In Section 8 we summarize the results.

## 2 Determination of the model

Now we consider an observer placed at the symmetry center of the spherically symmetric Stephani Universe ( $r \approx 0$ ). All our physical assumptions will concern his neighbourhood.

First of all we assume that locally, matter filling up the Universe fulfills a barotropic equation of state of the standard form

$$p(r \approx 0, \tau) = \alpha \rho(\tau), \quad (6)$$

where  $\alpha$  is some constant ( $\alpha \geq -1$ ). For example  $\alpha = 0$  corresponds to dust and  $\alpha = 1/3$  to relativistic matter. For negative  $\alpha$  pressure becomes negative and we regard an appropriate matter to be in exotic form. An extreme value of  $\alpha$  ( $\alpha = -1$ ) corresponds to cosmological constant.

Substituting (4) and (5) into (6) we find an explicit form of the function  $C(\tau)$

$$C(\tau) = \frac{A}{R^{3(1+\alpha)/2}(\tau)}, \quad (7)$$

where  $A$  is some constant. Choosing a curvature function  $k(\tau)$  in a form

$$k(\tau) = \beta R(\tau) \quad (8)$$

determines uniquely the function  $F(\tau)$ , which now reads

$$F(\tau) = \frac{1}{c} \frac{R^{3(1+\alpha)/2}(\tau)}{\sqrt{A^2 - \beta R^{2+3\alpha}(\tau)}}. \quad (9)$$

$\beta$  is a constant, which can be either negative, zero or positive.

The condition (8) is of course not the most general one. The simplicity was the only criterion for choosing it. However, one can easily show that any condition of the form  $k(\tau) = \beta R^\gamma(\tau)$  ( $0 < \gamma < 2$ ) gives similar results to that ones obtained in the present paper.

Now we are left with the time parameter  $\tau$  which is not uniquely determined so far. In FRLW models the time parameter is usually chosen globally to be a proper time of any comoving observer. In spherically symmetric Stephani models usually different choice is realized. Since there exists only one symmetry center in the models it seems natural to distinguish just this point. Hence the time parameter is chosen in a special way to be a proper time of an observer placed at the symmetry center. It is not a proper time of any other observer. To do that we put [16]

$$F(\tau)R(\tau)\frac{d}{d\tau}\left[\frac{V(r,\tau)}{R(\tau)}\right]=1, \quad (10)$$

what together with (8) and (9) gives a dynamical equation for the Stephani version of the scale factor  $R(t)$

$$\left(\frac{dR(t)}{cdt}\right)^2 + \beta R(t) = \frac{A^2}{R^{1+3\alpha}(t)}, \quad (11)$$

or more familiar form resembling a Friedmann equation

$$\left(\frac{dR(t)}{cdt}\right)^2 + k(t) = \frac{8\pi G}{3c^4}\rho(t)R^2(t), \quad (12)$$

where the energy density  $\rho(t)$  is calculated from (4)

$$\rho(t) = \frac{C_\alpha}{R^{3+3\alpha}(t)}, \quad (13)$$

and  $C_\alpha$  is some constant. New time parameter is now denoted by  $t$  and will be regarded as a cosmic time. Solution of the equation (11) could give us a full information about evolution of the model. What we are interested in is how the type of evolution is influenced by the numerical values of parameters of  $\alpha$  and  $\beta$ . Note that Barrett and Clarkson [15] also discussed two parameter class of spherically symmetric Stephani models but of different type. Their models do not admit dust-like ( $\alpha = 0$ ) equation of state at the symmetry center.

### 3 Classification of models

Qualitative classification of spherically symmetric cosmological models depending on different values of parameters  $\alpha$  and  $\beta$  can be performed without

solving the equation (11). For this purpose we use a standard method [17, 18] consisting in treating the evolution equation (11) as the energy conservation principle for some dynamical system with kinetic energy  $(dR/cdt)^2$  and potential energy

$$V(R) = \beta R - \frac{A^2}{R^{1+3\alpha}}. \quad (14)$$

Then the energy conservation principle reads

$$\left(\frac{dR}{cdt}\right)^2 + V(R) = 0. \quad (15)$$

Now, illustrating the potential energy  $V(R)$  on a diagram and keeping in mind that  $(dR/cdt)^2$  must be positive we deduce qualitative behaviour of the scale factor  $R(t)$  for different values of  $\alpha$  and  $\beta$ . We do not consider the case  $\beta = 0$  since it corresponds to FRLW Einstein-de Sitter model. All other models split naturally into two cases depending on the sign of the  $\beta$ -parameter. Moreover in the frame of each case five subcases can be pointed out, corresponding to the following ranges of the  $\alpha$ -parameter:

- a)  $\alpha > -1/3$ ,
- b)  $\alpha = -1/3$ ,
- c)  $-2/3 < \alpha < -1/3$ ,
- d)  $\alpha = -2/3$ ,
- e)  $-1 \leq \alpha < -2/3$ .

Appropriate behaviours of the potential energy  $V(R)$  and corresponding evolution of the scale factor  $R(t)$  are illustrated in Figs 1 and 2.

Some of the above presented cosmological models were already discussed in detail in literature. Especially much attention was paid to the model corresponding to  $\alpha = -1/3$  (Figs 1b, 2b) due to its particular simplicity (scale factor can be calculated explicitly) [16, 19]. In fact, some other models can also be expressed in an analytic way. In the present paper, however, we shall concentrate on the case, which mathematically is not so simple, but from physical point of view is very interesting because it relates to the SNIa observations suggesting accelerated expansion of the Universe at the present epoch [1]. The model we want to discuss is presented in Fig. 2a.

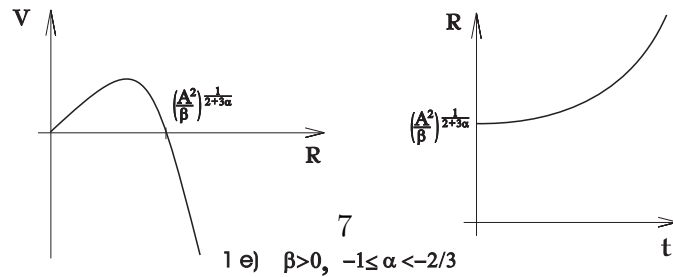
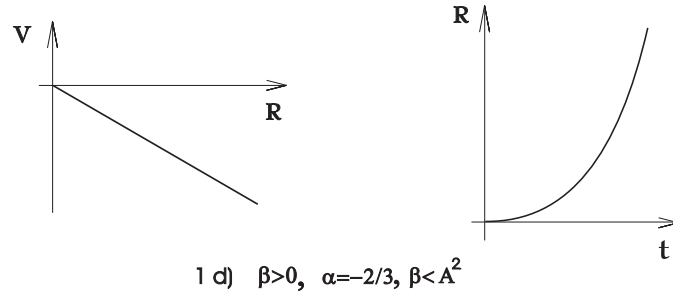
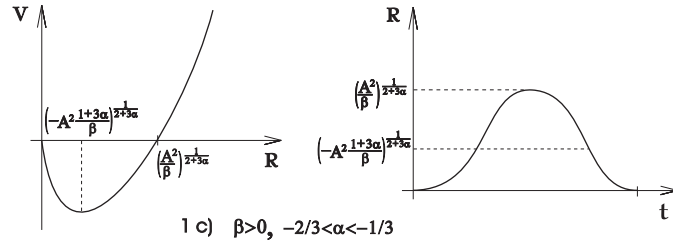
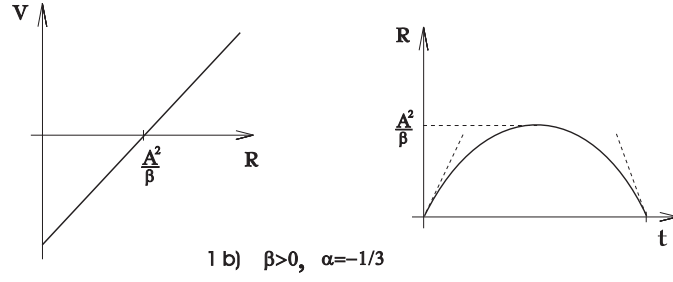
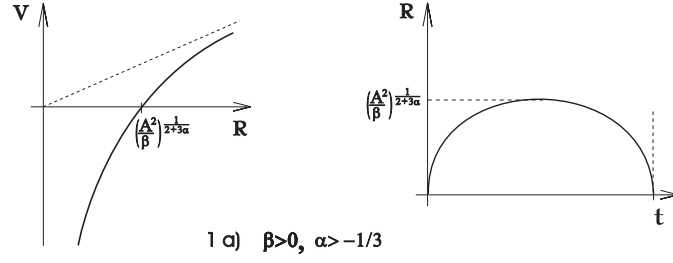


Figure 1: Potential energy  $V(R)$  of associated dynamical system (left diagrams) and corresponding evolution of the scale factor  $R(t)$  (right diagrams) for positive  $\beta$ -parameter (closed geometry) and different values of  $\alpha$ -parameter.

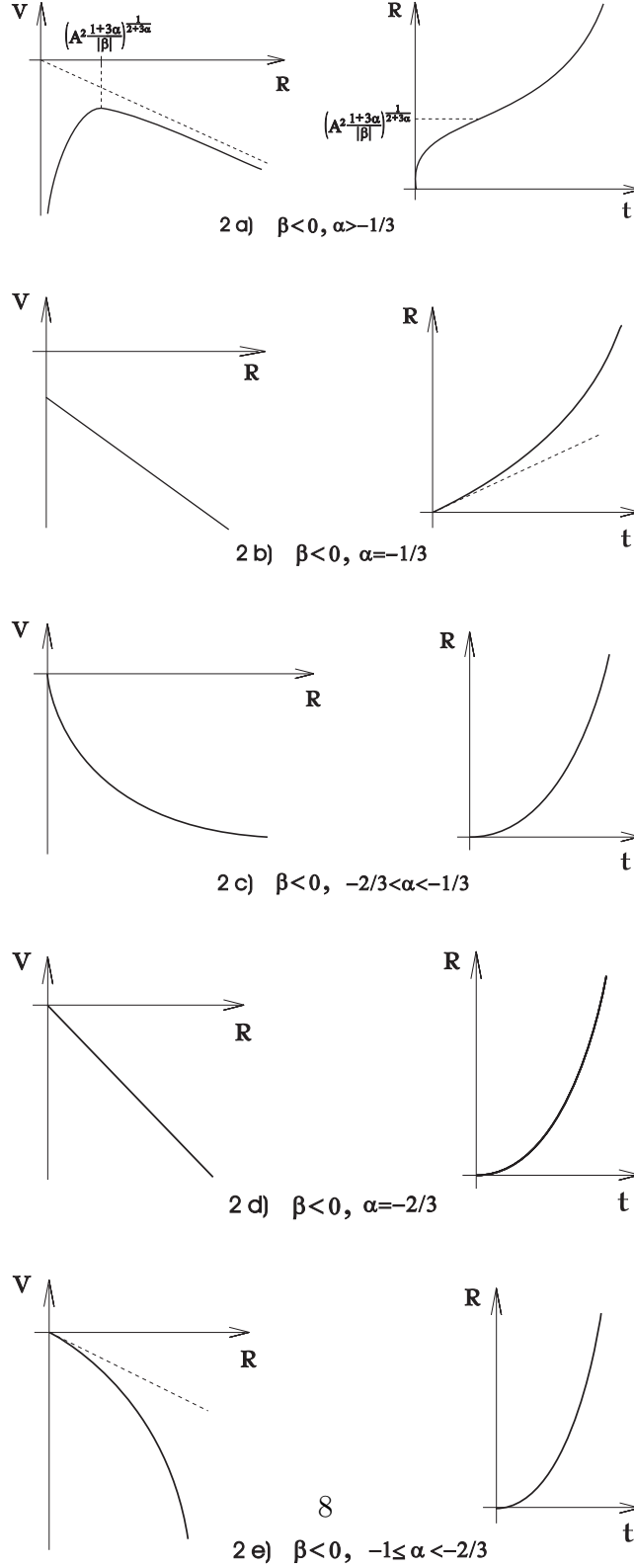


Figure 2: Potential energy  $V(R)$  of associated dynamical system (left diagrams) and corresponding evolution of the scale factor  $R(t)$  (right diagrams) for negative  $\beta$ -parameter (open geometry) and different values of  $\alpha$ -parameter.



## 4 Accelerating Universe

It is well known that in FRLW cosmology accelerated expansion of the universe may be only obtained in the case when the universe is filled with some exotic form of matter influencing negative pressure, e.g. cosmological constant. However, if we look at Fig. 2a we notice that in the spherically symmetric Stephani model the universe can exhibit accelerated expansion also in the case of vanishing or positive pressure at least at the symmetry center. Strictly speaking accelerated expansion takes place for every value of  $\alpha$  at least at later stages of evolution, provided that  $\beta < 0$  (hyperbolic geometry). But it seems to us that the model corresponding to vanishing  $\alpha$  is the simplest one, moreover it describes locally (in the neighbourhood of the observer located at the symmetry center) pressureless matter-dominated universe which is in agreement with local observations.

So putting  $\alpha = 0$  we find the energy density  $\rho(t)$  and the pressure  $p(r, t)$  to be equal to

$$\rho(t) = \frac{3c^4 A^2}{8\pi G} \frac{1}{R^3(t)} \quad (16)$$

and

$$p(r, t) = \frac{3c^4 A^2}{32\pi G} \beta r^2 R(t). \quad (17)$$

Note that indeed at the symmetry center ( $r \approx 0$ ) pressure vanishes, however, due to hyperbolic geometry  $\beta < 0$  it effectively becomes negative at larger distances from the symmetry center. So in spite of the fact that the energy density scales with the expansion as ordinary nonrelativistic matter ( $\rho \propto R^{-3}$ ), due to negativeness of pressure, for  $r > 0$ , it is exotic. Stephani version of the Friedmann equation now reads

$$\left(\frac{dR}{cdt}\right)^2 + \beta R = \frac{A^2}{R} \quad (18)$$

and unfortunately cannot be solved in an elementary way. However in two extreme cases corresponding to small and large  $R$  exact solutions can be easily found. They are

$$R(t) = \left(\frac{3}{2}Ac\right)^{2/3} t^{2/3} \quad \text{for small } R, \quad (19)$$

and

$$R(t) = \frac{1}{4}c^2|\beta|t^2 \quad \text{for large } R. \quad (20)$$

In the first case the solution is of Einstein-de Sitter type, while in the second case the evolution is evidently inflationary of power law type. It means that in our model the universe starts as decelerating and finally ends up as accelerating one. In the simplest FLRW cosmological models with one component fluid filling up the universe such behaviour is not possible. In the present model, inspite of the fact that formally it is of one component type (dust-like matter) the curvature term is non-trivial (see the condition (8)) and simulates the existence of some exotic fluid driving the power-law inflation at later epoch.

In the next Section we discuss some observational parameters in the model.

## 5 Observational Parameters

Observational parameters which describe kinematics of the evolution of the universe are Hubble parameter  $H$  and deceleration parameter  $q$ . In FLRW models they are constant on time slices due to homogeneity and isotropy. It does not to be the case in spherically symmetric Stephani models, in which homogeneity does not take place.

Following Ellis [20] we define  $H$  and  $q$  in a general case by

$$H \equiv \frac{1}{l} \frac{dl}{d\tau}, \quad q \equiv - \left( \frac{dl}{d\tau} \right)^{-2} \left( \frac{d^2 l}{d\tau^2} \right) l, \quad (21)$$

where  $\tau$  is here proper time measured along the particle world line, and  $l$  is some representative length along the particle world-lines.

In FLRW cosmological models the above expressions go over into the standard ones

$$H = \frac{\dot{R}}{R}, \quad q = - \frac{\ddot{R}R}{\dot{R}^2}, \quad (22)$$

where  $R$  is a scale factor entering the space part of the line-element

$$ds^2 = c^2 d\tau^2 - R^2(\tau) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right). \quad (23)$$

In our spherically symmetric Stephani models the line element is given by

$$ds^2 = \frac{c^2 dt^2}{V^2(r, t)} - \frac{R^2(t)}{V^2(r, t)} (dr^2 + r^2 d\Omega^2). \quad (24)$$

Hence neither  $t$  is proper time of any comoving observer, nor  $R(t)$  is a true scale factor (uniquely determining the length  $l$ ). In order to calculate  $H$  and  $q$  we have to rewrite the line element using global time parametrization

$$ds^2 = c^2 d\tau^2 - R^2(r, \tau) (dr^2 + r^2 d\Omega^2), \quad (25)$$

where we defined

$$d\tau \equiv \frac{dt}{V}, \quad R(r, \tau) \equiv \frac{R(t(r, \tau))}{V(r, t(r, \tau))}. \quad (26)$$

Now the definitions for  $H$  and  $q$  read

$$H = \frac{1}{R(r, \tau)} \frac{dR(r, \tau)}{d\tau}, \quad q = -\frac{1}{H^2 R(r, \tau)} \frac{d^2 R(r, \tau)}{d\tau^2}. \quad (27)$$

Note that generally  $H$  and  $q$  are not spatially constant anymore, what should not be surprising in non-homogeneous universe. It can be shown, however, that it is possible to choose time parameter with the aid of which defined Hubble parameter does not depend on  $r$ . It turns out that the time parameter  $t$  in the line element (24) is just such a parameter, i.e.

$$H = \frac{1}{R(r, \tau)} \frac{dR(r, \tau)}{d\tau} = \frac{1}{R(t)} \frac{dR(t)}{dt}. \quad (28)$$

In other words with this time parameter the expansion is the same at every point of time slice of the model. Unfortunately it does not happen in the case of  $q$  parameter which is evidently coordinate dependent [21]. Explicit calculation gives

$$q(r, t) = -V(r, t) \frac{\ddot{R}(t)}{R(t)H^2} + V(r, t) - 1 = -V(r, t)q(t) + V(r, t) - 1 \quad (29)$$

or

$$q(r, t) = q(t) + \frac{1}{4}\beta r^2 R(t)(1 + q(t)), \quad (30)$$

where we defined

$$q(t) = -\frac{\ddot{R}(t)}{R(t)H^2(t)}. \quad (31)$$

for deceleration parameter in neighbourhood of the symmetry center.

Since  $\beta < 0$  in our model, the deceleration parameter  $q(r, t)$  decreases with increasing distance to the observed galaxy. Physically it means that in our non-homogeneous universe acceleration of the expansion increases with distance.

In order to see how the deceleration parameter  $q(r, t)$  depends on the redshift we express  $\beta$ ,  $r$  and  $R(t)$  in terms of observational parameters  $H_0$ ,  $\Omega_0$  and  $z$ , where  $\Omega_0$  is energy density parameter at present epoch and  $z$  is a redshift of a galaxy placed at some point determined by comoving coordinate  $r$ . Since in the considered Stephani models energy density  $\rho(t)$  is constant in space, energy density parameter  $\Omega$  can be defined similarly as in FLRW models, i.e.

$$\Omega(t) = \frac{\rho(t)}{\rho_{cr}(t)}, \quad (32)$$

where

$$\rho_{cr}(t) \equiv \frac{3c^2}{8\pi G} H^2(t) \quad (33)$$

is critical energy density defined as actual energy density in the model with flat geometry ( $\beta = 0$ ).

Expressing the comoving coordinate  $r$  in terms of observational parameters is essential for our purposes and can be performed in a standard way by putting the infinitesimal space-time interval  $ds^2$  connecting two light events equal to zero (we choose coordinate system in such a way that  $\theta$  and  $\phi$  are constant along light ray path from galaxy to observer). Hence we get

$$cdt = -R(t)dr \quad (34)$$

or equivalently

$$\frac{R^{(3\alpha-1)/2} dR}{\sqrt{A^2 - \beta R^{2+3\alpha}}} = -dr \quad (35)$$

after getting rid of  $t$  variable using (11). Constants  $\beta$  and  $A^2$  can be found by writing down Friedmann equation (11) or (12) for the present epoch ( $t = t_0$ ) and by making use of the definition of  $\Omega_0$  parameter:

$$\beta = \frac{1}{c^2} R_0 H_0^2 (\Omega_0 - 1), \quad (36)$$

$$A^2 = \frac{1}{c^2} R_0^{3+3\alpha} H_0^2 \Omega_0. \quad (37)$$

Substituting these expressions into (35), next integrating both sides and changing integration variable we get integral formula for  $r$

$$r = \frac{c}{R_0 H_0} \int_{R/R_0}^1 \frac{x^{(3\alpha-1)/2} dx}{\sqrt{\Omega_0 + (1 - \Omega_0)x^{2+3\alpha}}}. \quad (38)$$

$R$  is a scale factor at the epoch when the light ray was emitted by the observed galaxy. Unfortunately integration cannot be performed explicitly either in general case or even in the special case corresponding to  $\alpha = 0$ . We compute it approximately assuming that  $\Omega_0$  is small with respect to the ratio  $R/R_0$ . Physically it means that in order to keep the approximation valid, the larger distances  $r$  are calculated, the smaller energy density parameter  $\Omega_0$  has to be taken into account. Then we skip the first term under the square root in (38) and the integration can be carried out explicitly giving

$$r = \frac{2c}{R_0 H_0 \sqrt{1 - \Omega_0}} \left( \sqrt{\frac{R_0}{R}} - 1 \right). \quad (39)$$

Together with the relation [16]

$$\frac{R_0}{R} = \frac{z + 1}{V(r, t)} \quad (40)$$

we obtain system of algebraic equations which can be solved with respect to  $R$  and  $r$

$$R = \frac{4R_0}{(z + 2)^2}, \quad (41)$$

$$r = \frac{cz}{R_0 H_0 \sqrt{1 - \Omega_0}}. \quad (42)$$

Substituting (28a) and (33) into (24b) we find present value ( $t = t_0$ ) of the deceleration parameter of the region of the universe where a galaxy with the observed redshift  $z$  is placed

$$q(z, t_0) = q_0 - \frac{1}{4}(1 + q_0)z^2. \quad (43)$$

$q_0$  is a local value of the deceleration parameter.

In this model, in spite of the fact that local observations of galaxies give positive value of  $q_0$ , in the case of more distant ones ( $z > \sqrt{q_0/(1+q_0)}$ ) negative deceleration parameter is measured.

More detailed description of the model from the point of view of observations requires derivation of coordinate independent relation between observational parameters. Redshift-magnitude relation is just such a relation and finding it will be our next task.

## 6 Redshift-magnitude relation

In cosmological models which are not homogeneous and isotropic one of the methods allowing for finding redshift-magnitude relation is based on formalism proposed by Kristian and Sachs [22] and Ellis and MacCallum [23] leading to relation in form of power series around the observer's position. But this method is not very effective if large redshifts are taken into account.

Due to spherical symmetry of the model it is possible, however, to find closed formula for apparent bolometric magnitude  $m$  of galaxy, as a function of redshift  $z$  in the case of small  $\Omega_0$ -parameter. The problem consists then in explicit calculation of the luminosity distance  $D_0$ .

The formula for the luminosity distance in our spherically symmetric model is the same as in FLRW models and reads [24]

$$D_0 = R_0 r(z+1). \quad (44)$$

Making use of earlier derived expressions for  $\beta$ ,  $r$  and  $R$  we find

$$D_0 = \frac{c}{H_0} z \frac{(z+2)^2}{4}. \quad (45)$$

For small redshifts ( $z \ll 1$ ) the expression goes over into usual distance-redshift relation known from Friedmann cosmology (we remind that the above formula is valid only for small values of  $\Omega_0$ )

$$D_0 \approx \frac{c}{H_0} z. \quad (46)$$

However, for larger redshifts it grows faster than in FLRW models, where

$$D_0 = \frac{2c}{H_0 \Omega_0^2} [\Omega_0 z + (\Omega_0 - 2)(\sqrt{\Omega_0 z + 1} - 1)], \quad (47)$$

and is in favor of SnIa observations.

Finding general formula for  $D_0$  requires numerical integration of (38). Before doing that we expand  $D_0(z)$  in our model into Taylor series with respect to the  $z$ -variable. Up to the second order we get (for arbitrary  $\Omega_0$  and  $\alpha$ )

$$D_0(z) = \frac{c}{H_0} \left\{ z + \frac{z^2}{4} [4 - 3\Omega_0(\alpha + 1)] \right\}. \quad (48)$$

Comparing with the analogous expansion of the Mattig formula [25], which up to the same order in the dust filled universe ( $\alpha = 0$ ) reads  $D_0(z) = (c/H_0)[z + (2 - \Omega_0)z^2/4]$ , we notice that for arbitrary  $\Omega_0$  redshift-magnitude formula in the Stephani model indeed grows faster than in the Friedmann model at least for relatively small values of  $z$ , for which the Taylor expansion is valid.

Since supernovae of type Ia favoring accelerated evolution of the Universe were discovered at relatively large redshifts  $z > 0.5$  full formula for the luminosity distance is required valid not only for small  $z$ . Hence Taylor expansion calculated up to low order is not satisfactory and we must calculate  $D_0(z)$  numerically.

The results of this numerical calculation are presented in figures 3 and 4. First (figure 3), we compare the redshift magnitude relation obtained in both Friedmann and Stephani spherically symmetric models determined by the same cosmological parameters  $H_0, \Omega_0$  and the  $\alpha$ -parameter responsible for the form of matter filling locally the universe. We realize that for relatively large redshifts the curves  $m(z)$  in both models diverge and the relation  $m(z)$  in the Stephani model looks like the appropriate relation in the Friedmann model but with the cosmological constant. In other words nonhomogeneity in the spherical symmetric Stephani model in some sense simulates existence of the  $\Lambda$ -term.

In Fig. 4 the redshift-magnitude relation in the Stephani model is compared with some observational data from the Supernova Cosmology Project [26] (see also [27]). We notice that the most distant supernovae of type Ia fit quite well to the dust-like ( $\alpha = 0$ ) Stephani cosmological model with the energy density parameter  $\Omega_0 \leq 0.3$ . The values of the Hubble constant  $H_0$  and the absolute magnitude  $M$  are the same as in [26, 28].

In the next section we compare numerically the age of the universe in the considered model and in FLRW cosmological models corresponding to the same value of  $H_0$  and  $\Omega_0$  parameters.

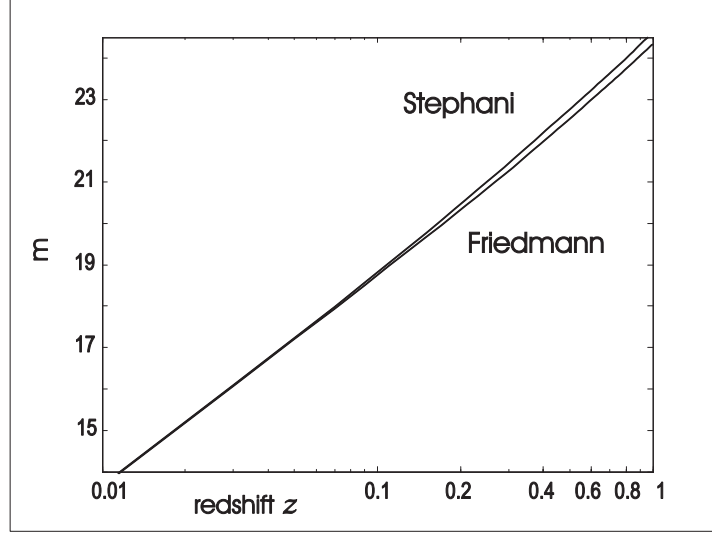


Figure 3: Redshift-magnitude relation  $m(z)$  in Friedmann cosmological model (lower line) and in the spherically-symmetric Stephani model (upper line). In both models the same cosmological parameters are chosen:  $H_0 = 65$  km/sMpc,  $M = -19.5$ ,  $\Omega_0 = 0.3$ .

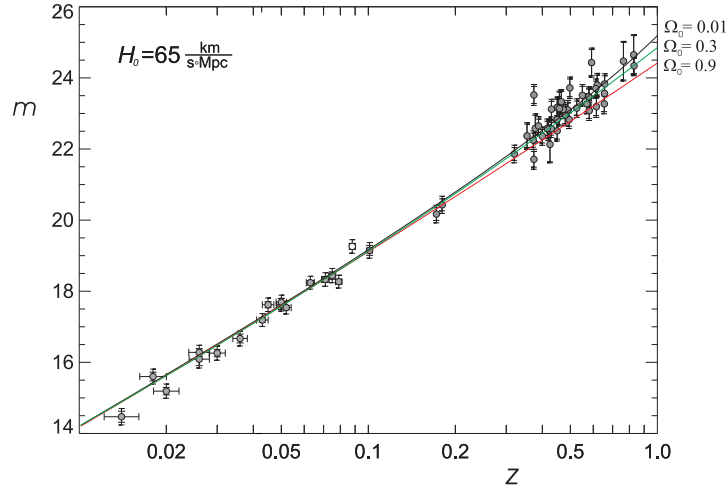


Figure 4: Redshift-magnitude relation  $m(z)$  fitted to observational data from the Supernova Cosmology Project.  $H_0 = 65$  km/sMpc,  $M = -19.5$ ,  $\Omega_0 = 0.3$ .



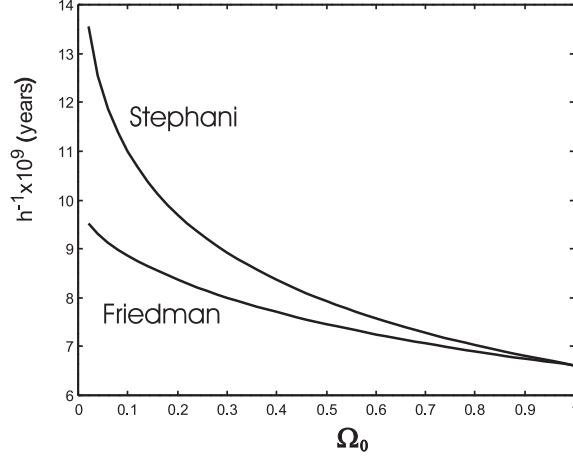


Figure 5: Age of the universe in spherically symmetric Stephani cosmological model (upper line) and in FLRW cosmological model (lower line) as a function of energy density parameter  $\Omega_0$ . It is assumed that  $\alpha = 0$  and  $H_0 = 100h$  km/sMpc, where  $1/2 \leq h \leq 1$ .

## 7 Age of the universe

We find the age of the universe in the considered models by direct integration of Stephani version of the Friedmann equation (11)

$$t_0^S = \frac{1}{c} \int_0^{R_0} \frac{R^{(1+3\alpha)/2} dR}{\sqrt{A^2 - \beta R^{2+3\alpha}}} = \frac{1}{H_0} \int_0^1 \frac{x^{(1+3\alpha)/2} dx}{\sqrt{\Omega_0 + (1 - \Omega_0)x^{2+3\alpha}}}. \quad (49)$$

Analogously calculated age of the universe in FLRW models reads

$$t_0^F = \frac{1}{H_0} \int_0^1 \frac{x^{(1+3\alpha)/2} dx}{\sqrt{\Omega_0 + (1 - \Omega_0)x^{1+3\alpha}}}. \quad (50)$$

We easily realize that the age in Stephani models is longer than in FLRW ones corresponding to the same values of the parameters  $H_0$ ,  $\Omega_0$  and  $\alpha$ . For  $\alpha = 0$  and  $H_0 = 100h$  km/sMpc ( $1/2 \leq h \leq 1$ ) and on the same diagram (Fig. 5) we plot the age of the universe,  $t_0^S$  and  $t_0^F$ , in both models as a function of energy density parameter  $\Omega_0$ .

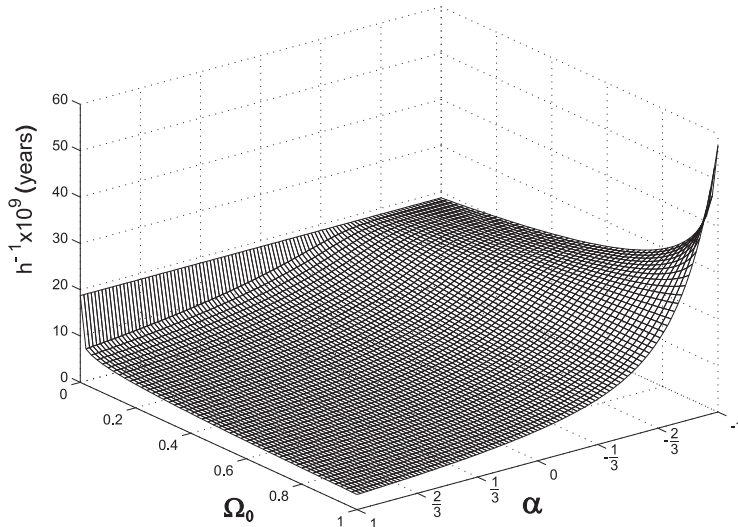


Figure 6: Age of the universe in spherically symmetric Stephani cosmological model as a function of  $\alpha$  and  $\Omega_0$ .

In Fig. 6 we plot the age  $t_0^S(\alpha, \Omega_0)$  of the Universe in Stephani model as a function of two parameters:  $\alpha$  – determining equation of state at the symmetry center, and  $\Omega_0$  – energy density parameter. We notice that the age increases with decreasing both  $\alpha$  and  $\Omega_0$  (provided that  $\alpha \geq -2/3$ ). In Fig. 7 we present sections of the plot  $t_0^S(\alpha, \Omega_0)$  corresponding to the values of  $\alpha = -2/3, -1/3, 0, 1/3, 2/3, 1$ .

## 8 Conclusions

In the paper we considered a class of spherically symmetric Stephani cosmological models parametrized by two parameters  $\alpha$  and  $\beta$ . The  $\alpha$ -parameter determines the form of barotropic equation of state in the neighbourhood of the symmetry center of the models and  $\beta$ -parameter is related to the curvature index. We performed qualitative classification of evolution types of models depending on different values of  $\alpha$  and  $\beta$ . We showed that for negative  $\beta$  the universe exhibits accelerated expansion independently of  $\alpha$ , what does not take place in FLRW models. We focused our attention on the model corresponding to  $\alpha = 0$ . In this model pressure is negligible in the neighbour-

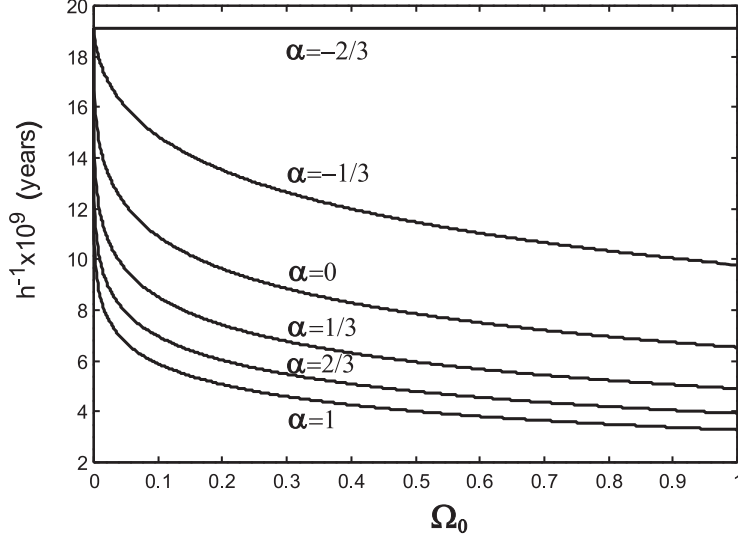


Figure 7: Age of the universe in spherically symmetric Stephani cosmological model as a function of  $\Omega_0$  for different values of  $\alpha = -2/3, -1/3, 0, 1/3, 2/3, 1$ .

hood of the symmetry center (the so-called dust-like model) and the model looks like of Einstein-de Sitter type at the early stage of evolution. At later epoch the evolution is inflationary of power law form. This result seems to be interesting because we showed that existence of “exotic” matter (influencing negative pressure) is not necessary (at least locally) to drive accelerated expansion of more distant regions. We also showed that acceleration becomes larger while increasing the distance to the galaxy.

We derived redshift-magnitude formula in the model and showed that for large relatively large redshifts ( $z \approx 1$ ) apparent bolometric magnitude  $m(z)$  grows faster than in corresponding FLRW model what is in favor of SNIa observational data. We found numerically that a good fit is obtained for  $H_0 = 65$  km/sMpc and  $\Omega_0 \leq 0.3$ . At this point it should be noted that the good fit of  $m(z)$  to the SNIa observational data in spherically-symmetric Stephani cosmological model has been already obtained by Dąbrowski and Hendry [28]. However in their model they assumed a string-like ( $\alpha = -1/3$ ) equation of state at the center of symmetry.

Finally we found that the age of the universe in the considered model is

remarkably larger than in isotropic and homogeneous models.

Concluding we would like to stress that we do not claim that our surrounding universe is of Stephani type. We wanted to show that there exist different from isotropic and homogeneous cosmological models which give the same observational evidencies as FRWL models with the cosmological constant. In these models the deviation from homogeneity could be so small that locally (up to several hundreds of megaparsecs) it would be undetectable. However its existence could show up at larger distances.

In the paper we considered only one fluid component filling up the universe. It is of course oversimplification and is hardly acceptable especially in the context of negativity of pressure at larger distances. It can be shown, however, that adding usual dust component into the model do not change relations (16, 17) significantly (only a constant in (16) will be different) and the qualitative result will be the same.

It seems that these two different fluid components should effect in the form of the power spectrum of the CMB anisotropies. Calculation of this power spectrum in the proposed model will be our next task.

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## References

- [1] Riess G *et al* 1998 *Astron. J.* **116** 1009
- [2] Vilenkin A 1984 *Phys. Rev. Lett.* **53** 1016
- [3] Davies R L 1997 *Phys. Rev. D* **36** 997
- [4] Silveira V and Waga I 1994 *Phys. Rev. D* **50** 4890
- [5] Kamionkowski M and Toumbas N 1996 *Phys. Rev. Lett.* **77** 587
- [6] Caldwell R R, Rahul D and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1582
- [7] Rudnicki K 1995 *The Cosmological Principle* (Kraków: Jagiellonian University, ed. J. Maśłowski)
- [8] Ellis G F R, Maartens R and Nel S D 1978 *Mon. Not. Roy. Astr. Soc.* **184** 439

- [9] Stephani H 1967 *Commun. Math. Phys.* **4** 137
- [10] Stephani H 1967 *Commun. Math. Phys.* **5** 337
- [11] Kramer D *et al* 1980 *Exact Solutions of Einstein's Field Equations* (Cambridge: Cambridge University Press)
- [12] Krasiński A 1983 *Gen. Rel. Grav.* **15** 673
- [13] Krasiński A 1997 *Inhomogeneous Cosmological Models* (Cambridge: Cambridge University Press)
- [14] Dąbrowski M P 1993 *J. Math. Phys.* **34** 1447
- [15] Barrett R K and Clarkson C A 2000 *Class. Quantum Gravity* **17** 5047
- [16] Dąbrowski M P 1995 *Astrophys. J.* **447** 43
- [17] Coquereaux R and Grossmann A 1982 *Ann. Phys. N.Y.* **143** 296.
- [18] Dąbrowski M P and Stelmach J 1986 *Ann. Phys. N.Y.* **166** 422
- [19] Dąbrowski M P 1999 *Preprint* gr-qc/9905083
- [20] Ellis G F R 1973 *Cargèse Lectures in Physics*, Vol. 6, ed. E Schatzman, Gordon and Breach Science Publ., p.1
- [21] Sussman R A 1999 *Preprint* gr-qc/9908019
- [22] Kristian J and Sachs R K 1966 *Astrophys. J.* **143** 379
- [23] Ellis G F R and MacCallum M A H 1970 *Comm. Math. Phys.* **19** 31
- [24] Partovi M H and Mashhoon 1984 *Astrophys. J.* **276** 4
- [25] von Mattig W 1958 *Astron. Nachr.* **284** 109
- [26] Perlmutter S *et al* 1998 *Astrophys. J.* **516**
- [27] Hamuy M *et al* 1996 *Astron. J.* **112** 2391
- [28] Dąbrowski M P and Hendry M A 1998 *Astrophys. J.* **498** 67